

Un demi-siècle des RdPs



- Un demi-siècle du travail définissant un point de repère dans la théorie des systèmes dynamiques à événements discrets (SDEDs) mérite d'être souligné.
- On ne prétend que faire une modeste commémoration de sa vie et vision, et présenter des traces sur la discipline créé, sachant que la science et la technique sont des constructions sociales.
- Manuel Silva: "Half a century after Carl Adam Petri's Ph.D. thesis: A perspective on the field", Annual Reviews in Control 37:2, 191-219, 2013.

Online publication date: 1-Dec-2013.

Contents of this talk



- 1. Introduction
- 2. About Carl Adam Petri
- 3. Petri Nets: On Basic Properties and ways of approach
- 4. Petri Nets: A modeling paradigm
- 5. On five interactions over twenty five years

Carl Adam Petri: Half a century after his PhD



When in **Computer Science** the paradigm was **local** computations on mathematically intricate problems...

- ... Carl Adam Petri look for a **Systems Theory**, far beyond what was the problematic in Computer Science, applicable in a broad landscape of research fields:
 - Automatic Control, Operations Research, Informatics...
 - Automation of manufacturing, transport and logistic systems, communications protocols, software and hardware engineering, (bio)chemistry, epidemiology, law/rights, demography...

Carl Adam Petri: Half a century after his PhD



Interested in the description of some real-life situations, as mathematician he essentially worked at *conceptual level*, "opening windows" (i.e., providing foundations for new ways of thinking), what may be viewed as a speciality less frequently exercised with success than "theorem prover".

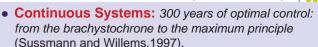
At the personal level he always was a warmhearted person

Science and Technology are social constructions



- Nevertheless, in their development some people contribute in outstanding ways.
- The recognition of their achievements is sometimes partially done by giving his/her name to some measurement unit, universal constant, algorithm, etc.
 - International System of Units (newton, coulomb, ampere, volt, farad...)
 - Coriolis, Young, Avogadro, Kirchoff, Minkowsky-Farkas, Donsker, Floyd, Dijikstra, Wiener, Kalman, BCMP...
 - Naming an entire subfield is much less frequent: Markov Chains, Hilbert Spaces, Galois Fields, Petri Nets...

Continuous ("natural") vs Discrete ("artificial")



- From logic/discrete automatisms to DEDS (Ho et al., 1980)
- Discrete Event System Specification (DEVS; simulation: Zeigler, 1976)

[after: SIMSCRIPT, SIMULA, GPSS...]

 Like Le Bourgeois Gentilhomme of Molière, users or developers on topics related to automata, Markov chains, queueing networks, Petri nets, process algebras, max/plus algebra, etc., realized that we were speaking not in prose, but "in DEDS". ©

Continuous ("natural") vs Discrete ("artificial")



 In Future directions in control theory: a mathematical perspective (Fleming, 1988):

there exist no formalisms for DEDS mathematically as compact or computationally as tractable as the differential equations are for continuous systems.

- "Natural" selection (Darwinism...), but NOT monism...
- But, ideas organized in modeling paradigms

PNs: an option (more than a question of taste, but...)

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Carl Adam Petri (Leipzig,1926-Bonn, 2010)



- His father, professor of mathematics, enjoys friendly relationships with mathematicians as David Hilbert and Hermann Minkowski.
- Obliged to join the German army (1944), he becomes prisoner in England (till 1946), where later he teaches Latin and chemistry.
- Petri returns to Germany in 1949, studying mathematics at the Technical University of Hannover (till 1956);
- From 1959 until 1962 he works at the University of Bonn;
- In 1962 he gets the PhD degree from the Technical University of Darmstadt.
- Until 1968 Petri is working at the University of Bonn, becoming head of the computer installations.
- In 1968 was founded the GMD. He become later one of the directors of the Institute for Methodological Foundations.
- He retires from GMD in 1991, but in 1988 was named Honorary Professor at University of Hamburg.

Petri nets: When? Were? How?



- Against what it is frequently quoted, in his PhD dissertation the graphical notation on PNs does not appear!!!
- The well-known bipartite graphs with conditions/places and events/transitions will come some three years later.
- But he claims to invent
 - it in a playful mood in August 1939, and practiced it intensively for the purpose of memorizing chemical processes, using *circles for substances* and *squares for reactions*
- A. Holt (MIT, Project MAC) give his name to the bipartite nets
- Replacing temporal order by causal order, he looked concurrency as mutually causal independent occurrences (synchronization as rendez-vous)
- Causal orders and preservation laws were inspired to Petri by the laws of Physics

Robin Milner (Turing Award, 1991)



- In the 1970s, Robin Milner focusses his view of concurrent systems on the "interactions of smaller components", being one of the more celebrated steps in the definition of process algebras.
- In other words, Milner pays central attention to the process of construction of the model:

[Petri] pioneered the scientific modeling of discrete concurrent systems. Petri's work has a secure place at the root of concurrency theory...What I always wanted to advance, to complement Petri net theory, is the synthetic or compositional view of systems which is familiar from programming. (1993)

PNs & international meeetings

- Since 1980 it exists an annual conference devoted to PNs, the Int. Conf. on Application and Theory of Petri Nets and Concurrency (ICATPN; in fact, from 1980 until 1988 it was known as International Workshop, IWATPN).
- From 1985 till 2003, ten editions of the IEEE/ACM Int. Symp.
 on Petri Nets and Performance Models (PNPM) took place.
 This meeting merged in 2004 with others, as that on Process
 Algebra and Performance Models (PAPM), giving light to the
 annual Int. Conf. on Quantitative Evaluation of Systems
 (QEST).
- But what is more impressive is the number of meetings in which PNs is a topic explicitly mentioned, frequently devoting special sessions to them. For example, in "The Petri Nets: Meetings and Events" we can count more that 150 during 2005-2011, so more than twenty per year. (What is truly a lower bound!!!)

Among other important recognitions to Carl Adam Petri



- The Konrad-Zuse-Medal (1993);
- The Society for Design and Process Science established the Carl A. Petri Distinguished Technical Achievement Award (1997);
- The Werner-von-Siemens-Ring (1997), one of the highest ranked award for technical sciences in Germany, delivered every three years;
- The Doctorate Honoris Causa by the Universidad de Zaragoza, in 1999.
- The Order of the Dutch Lion, as Commander (2003);
- IEEE Computer Pioneer (2008):

For establishing Petri net theory in 1962, which not only was cited by hundreds of thousands of scientific publications but also significantly advanced the fields of parallel and distributed computing.

Most of the recognitions to Carl Adam Petri, a mathematician, came from engineering.

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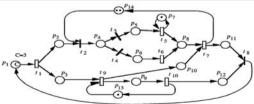
Petri Nets are operational formalisms (how the system works, vs what it does)

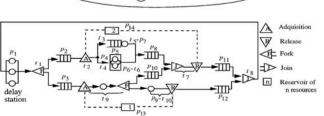


Among the qualities that they enjoy are:

- Minimality in the number of primitives (trade-off between the engineering and scientific perspectives).
- **Locality and structuration** (parallelism, synchronization, conflicts, etc.) >>>
 - Cooperation & Competition relationships.
 - Top-down & Bottom-up model building.
- Non-determinism (humble position---time unpredictability).
- Temporal realism and occurrence nets.

Minimality-understanding (theory) vs rich ontology-use (engineering)





... ways of approach to PNs



if all roads lead to Rome ...
more than "probably", Rome was/is important!

... to "the town" of Petri Nets (with several nuances) we may arrive using different **roads** ...:

- An axiomatic view (the C.A. Petri), +++
- Continuous System Theory, from state transition (...like in VAS)
- Theory of Regions: a kind of field-coding in labelled graphs (synthesis problem / "mon cher Philippe Darondeau"...)
- Non-monotonic logic: Linear logic, Girard / R. Valette

From 1950's to 1960's

Salient features:

- Bipartite --- dual objects!
- Locality --- compositionality!
- Timed vs untimed?
- Discrete vs fluid views?

		Reservoirs	Activities
1957	QNs	Queues/	Stations/
		Clients	Servers
1961	FDs	Deposits/	Valves/
		Levels	Flows
1962	PNs	Places/	Transitions/
		Conditions	Events

P/T systems constitute a "frontier formalism"



- P/T systems are able to model infinite state (i.e., unbounded) systems.
- Constitute a "frontier formalism":
 - Are very expressive and most of classical properties are decidable,
 - But "small" extensions transform it into a formalism able to simulate Turing machines, thus excellent expressivity, but plenty of undecidabilities.
 - Among the evoked extensions, just the addition of:
 - inhibitor (or zero test) arcs,
 - priority levels to the firing of transitions; or
 - certain time-interpretations.

One "children" of ordinary PNs: The Grafcet

- GRaphe de l'AFCET (Association Française de Cybernétique Économique et Technique)
- GRAphe (Fonctionnel) de Commande Etape-Transition
- "Like" an ordinary P/T net system in which a flipflop approach is used for implementing places, even if they are not 1-bounded; also problems with unresolved conflicts! (be careful because this destroy the tokens conservation laws!)

Sequential Function Chart (SFC)
Norms IEC 848 (1988) + IEC 61131-3 (1993)



LES AUTEURS DU RAPPORT Grafcet

Ce rapport est le fruit d'un travail collectif des 24 membres de la « commission de normalisation de la représentation du cahier des charges d'un automatisme logique » — Groupe de travail AFCET « Systèmes logiques » dont voici les noms :

BLANCHARD, ex-CERT-DERA-Tou-BOERI, LASSY-Nice BONS, USSI-Bagneux BORREL, Automobile Peugeot-Automobiles Montbéliard BRARD, La Télémécanique Électrique-Rueil DAVID, Lab. Automatique-Grenoble DESMOULINS, SEMS-Échirolles ESQUISSAUD, EDF-Chatou FRACHET, ISMCM-Saint-Ouen GIRARD. Carbone-Lorraine-Gennevilliers LEEMANS, Merlin Gerin-Grenoble MARIN. LASSY-Nice

MOALLA, IMAG-Grenoble

PEIRANO, C.E.A.-Cadarache POTET. EFCIS-Grenoble PRUNET, Lab.-Automatique-Montpellier ROUSSEAU. EFCIS-Grenoble SIFAKIS. IMAG-Grenoble SILVA SUAREZ, Lab. Automatique Grenoble THUILLIER, La Télémécanique Électrique-Carros TOULOTTE, C. Automatique-Lille TOURRES, EDF-Clamart VALETTE, LAAS-Toulouse ZAHND, E.P.F.-Lausanne Animateur du Groupe de travail AFCET Systèmes logiques » : Michel BLANCHARD.

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ISO norm (mainly) in the framework of software engineering.

- International Standard Organization (ISO)
- International Electrotechnical Commission (IEC)
- The ISO/IEC 15909, publicized into two parts:
 - (1) Systems and software engineering -High-level Petri nets - Part 1: Concepts, definitions and graphical notation, 2004;
 - (2) Systems and software engineering -High-level Petri nets - Part 2: Transfer format, 2011

Maturity & engineering disciplines

The necessity of **formal methods** is beyond question in mature engineering disciplines.

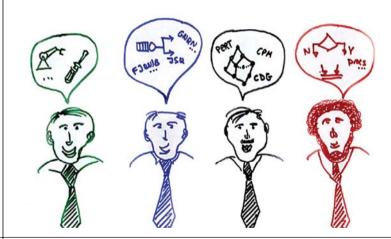
Maturity of an engineering discipline requires:

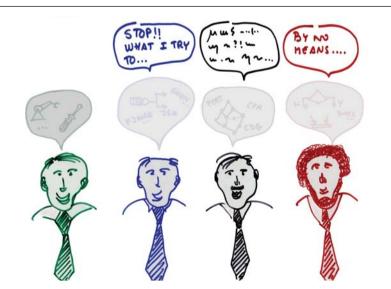
- Formalisms ("secure reasoning")
- Methodological support (easiness of use, reusability...)
- Existence of expertise (e.g., in terms of catalogs of models and methods for specific domains)
- Analysis techniques
- Synthesis procedures
- (Software) Tools
- Standards & international norms (ISO, IEC...)
- Implementation with different technologies (+ fault-tolerance, etc.)

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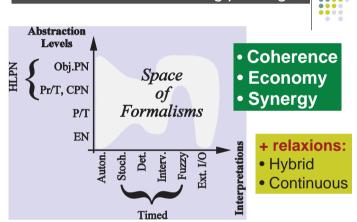




From a "formalism" vs a "modeling paradigm"

- Long life-cycle of many systems >>> family of related formalisms rather than a collection of "unrelated" formalisms.
- A paradigm is (Kuhn, 1962): [...] the total pattern of perceiving, conceptualizing, acting, validating, and valuing associated with a particular image of reality that prevails in a science
- A modeling paradigm is a conceptual framework that allows to obtain formalisms from some common concepts and principles.
- But multidisciplinarity!!!... also multiformalisms...

PNs constitute a modelling paradigm



An elementary example: Basic manufacturing cell & life-cycle

- Performance vs buffer capacity: monotonicity (?)
- Frequential view:

filtering effect due to the buffer

- Gantt diagrams: two scheduling strategies
- Marking diagram: control

or a branch of science.

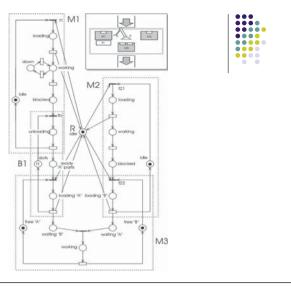
- Error detection & correction:
 - Duplication vs observation
 - Observable view

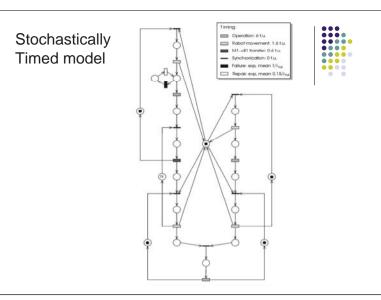
The basic manufacturing cell & life-cycle

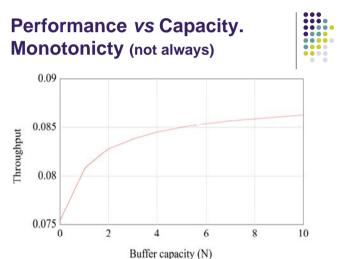
- Two processing machines (M1 and M2)
- An assembly machine (M3)
- A robot (for transportation purposes, R)
- A buffer (inventory, deposit,...) to uncouple upstream and downstream of M1
- Cooperation relationships (3 machines)
- Machine-buffer-machine: decoupling
- Competition (mutex-robot)

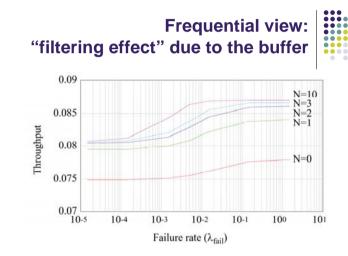


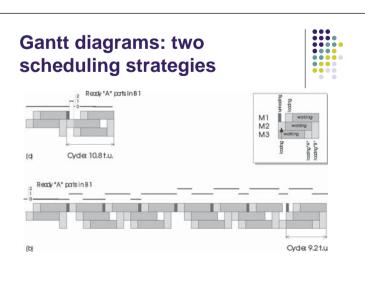


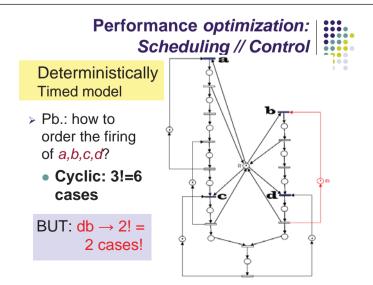


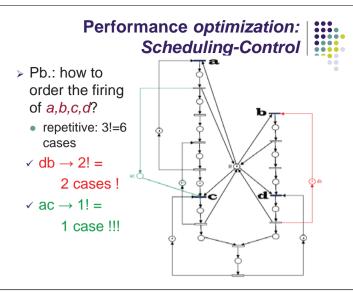


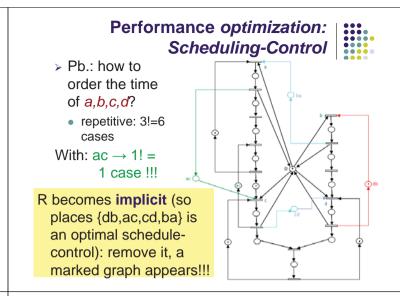


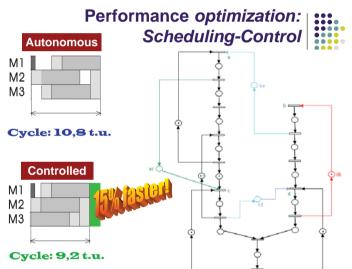


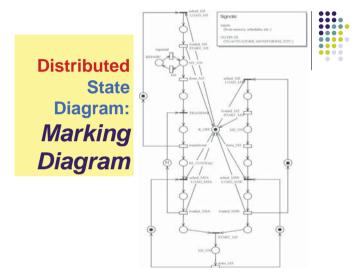


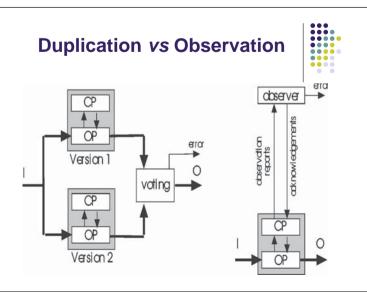


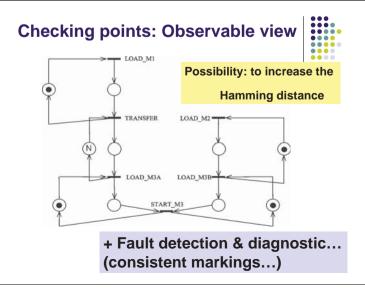












multiformalisms... multi-paradigmatic bridges

 even if we believe that PNs constitute an adequate conceptual framework or paradigm for the operational description of DEDS:

"we do not believe that it is always possible to select a single formalism, or family of them, to deal in a reasonable way with every aspect of every DEDS. The complexity and variety of systems suggest instead the interest of having multi-paradigm environments where the existence of sound and efficient bridges between different paradigms becomes a major issue" (Silva and Teruel, 1996).

In a different, but close context:

"I reject the idea that there can be a unique conceptual model, or one preferred formalism, for all aspects of something as large as concurrent computation." (Milner, 1993)

The important fact: the existence of inter-paradigmatic transformations





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5. On five interactions over twenty five years



In this part I will consider five points of contact with Carl Adam, over more than 25 years.

The first recall our fully disjoint set of interests at the given time (1976)

Synchronic relations and fluidization are two topics I sporadically discuss with him, Carl Adam showing big interest. Even sometimes we arrive to agree on "doing something together", but...

Just five points... over more than a quarter of century



- 1. A student in Grenoble, 1976
- 2. At the Bocconi University & Oxford, 1986
- 3. At he European Workshop on Petri Nets, Zaragoza, 1987
- 4. Carl Adam Petri, Doctor Honoris Causa by the University of Zaragoza, 1999
- 5. The International Conference on Petri Nets and Related Models, Miami, 2005

I. I was a PhD student in Grenoble. LAG (+ENSIMAG), 1975-1978



- The first time I meet C.A. Petri was at the **ENSIMAG** (Mme. Saucier, J. Sifakis...), 1976
- My interest was important because months before I abruptly change the topic of my PhD from:
 - interconnected automata & modular hardware synthesis (CUSAs / "hazards under control") to
 - Petri nets & programmable logic controllers (PLCs)
- The topic of his talk? --Synchronic distance related...?
- But at that period I deal with problems of quite different nature (software implementation issues, performance evaluation.../ he does not like explicit timing...)

Just five points... over more than a quarter of century

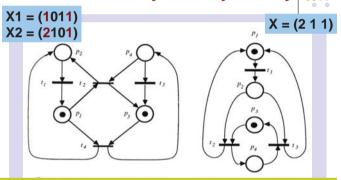
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II. At the Bocconi University & EWPN-Oxford, 1986



- The context (Bocconi Univ.: A. Pagnoni):
 - I was interested on transitions firing dependences in "generalized" (weigthed) PNs (P/T)
 - Petri (1975/76): Synchronic Distance, S-completion...
 - Other previous works by: J. Sifakis, U. Golz, I. Suzuki, T. Kasami, T. Murata...
 - My cooperation with T. Murata (USA-Spain Joint Com. for Scientific & Technological Research, 1985/88)
- The fact: existence on "Non-linear Synch. Relations"
- I did show Carl Adam one (the) "smallest" ordinary. pure, live & bounded net system...

II. At the Bocconi University & **EWPN-Oxford...: Synchrony Theory**



Synchrony Theory: a branch of General Net Theory devoted to the study of transitions firing dependences

II. At the Bocconi University & **EWPN-Oxford...: Synchrony Theory**



Linearly based synchronic properties: **Synchronic lead & Distance** A new way of measuring: SD is a metric

Basic finite delay synchronic properties:

B-deviation & B-fairness

Quantitative dependences (value) &

synchronic relations (boundedness)

Synchronic relations: behavioural (given m_o) & structural (for any m₀)

II. At the Bocconi University & **EWPN-Oxford...: Synchrony Theory**



- SD-relation ▶ BF-relation, the reverse does not hold
- Structural BF & SD relations (SBF; SSD):
 - are equivalence relations and
 - can be computed in polynomial time.
- In bounded systems, realizability of min-T-semiflows
 - ► Behavioural = Structural Synchronic relations

DUALITY:

T-views (T-semiflows) vs P-views (implicit places, eventually on partial nets)

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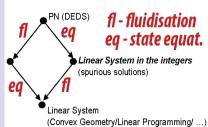
III. The European Workshop on Petri Nets, Zaragoza, 1987

- "On the computation of structural synchronic invariants in P/T nets", co-authored with J.M. Colom.
- The idea: systematic use of LPP, weak & strong duality theorems, unboudness theorems & alternatives theorems.
- Polynomial time computation of bounds ("like" fluid relaxation) of the synchronic values, and of the structural synchronic relations (lead, distance, deviation & B-fairness).
- Precedents on the use of:
 - LPP: H.J. Genrich & K. Lautenbach, for marked graphs (but exact values, because the incidence matrix is unimodular)
 - Farkas' lemma / alternatives theorems: G. Memmi & J.

III. The European Workshop on Petri Nets, Zaragoza, 1987



- Continuous Petri Nets
 - Explicitly (net level): R. David & H. Alla
 - Implicitly (relaxation at the state equation level): Systematic use of LPP
- A topic received with reluctances in the **EWPN / ICATPN (like** stochastic PNs were sometime before)



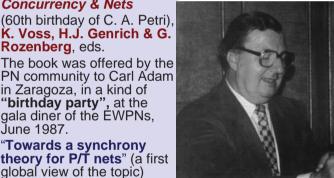
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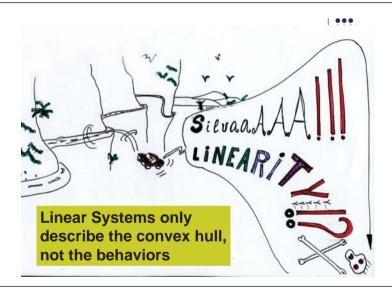
- consistency &
- · no-empty siphon at the initial marking)

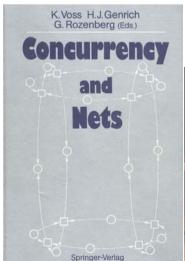




- on Petri Nets, Zaragoza, 1987 Concurrency & Nets (60th birthday of C. A. Petri),
- Rozenberg, eds. The book was offered by the PN community to Carl Adam in Zaragoza, in a kind of "birthday party", at the gala diner of the EWPNs, June 1987.
- "Towards a synchrony theory for P/T nets" (a first global view of the topic)







III. EWATPNs. Zaragoza, 1987





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IV. C.A. Petri, Doctor *Honoris Causa* by the University of Zaragoza, 1999

- The opportunity:
 - 25 years of the foundation of the Engineering School: 1974-1999
 - 10 years of the transformation in Polytechnic Faculty ("High School"): 1989-1999
 - The dean of Engineering: prof. Javier Martínez, the first PhD on Petri Nets at the University of Zaragoza
- The first three HC on engineering at Zaragoza

The day before (14-IV-99): an international seminar in his honor



- G. Balbo (Universitá di Torino):
 On PNs and Performance Evaluation: The GSPN case
- J. Billington (University of South-Australia):
 On the ISO/IEC Petri Net standard (15909)
- D. de Frutos (Universidad de Madrid):

 Decidable properties in Timed Petri nets
- M. Koutny (University of Newcastle upon Tyne):
 Combining Petri Nets and Process Algebras
- M. Silva (Universidad de Zaragoza):
 On continuous Petri Nets

IV. C.A. Petri, Doctor *Honoris Causa* by the University of Zaragoza, 1999



- Foundation of the University of Zaragoza:
 - from «Universitas magistrorum»
 - to «Universidad general de todas las ciencias»
 - Privilege of the Emperor & King Charles V, 1542
 - Papal Bull of Julius III, 1554.
- ... but "no funding", until some 40 years later...: Pedro Cerbuna. 1582.
- Engineering (E.T.S. Ingenieros Industriales: Mechanical & Electrical) & Economics: 1974
- As dean of Engineering, 1989: I obtain the transformation of the ETSII into Centro Politécnico Superior (+Telecommunication, + Informatics, + Chemical, ...)

IV. C.A. Petri, Doctor *Honoris Causa* by the University of Zaragoza, 1999



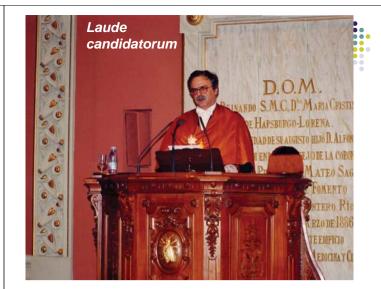
- The program was conceptually based on three basic pillars of technology, and three outstanding personalities:
 - Materials: Prof. Steve Tsai, pioneer in Composite Materials, from Stanford University.
 - Energy: Prof. Amable Liñán, pioneer in the fluid

 mechanics perspective for Combustion
 Engineering, from the Polytechnic University of Madrid.
 - Information: Prof. Carl Adam Petri, as representative of System Theory & Computer Science, Univ. of Hamburg (ex-GMD, Bonn).

The entourage: towards the Paraninfo

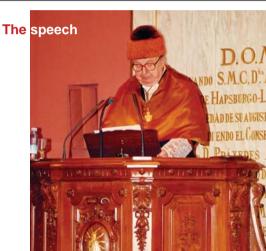












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- 2. At the Bocconi University & Oxford, 1986
- 3. The European Workshop on Petri Nets, Zaragoza, 1987
- 4. Carl Adam Petri, Doctor *Honoris Causa* by the University of Zaragoza, 1999
- 5. The International Conference on Petri Nets and Related Models, Miami, 2005

V. The Int. Conf. on Petri Nets and Related Models, Miami, 2004

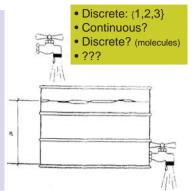


- Context for fluid models:
 - System Theory
 - Automatic Control Theory (observability, controllability...)
 - Operations Research
 - Scalability of models with "large populations" (QNs, Compartmental Systems, SFGs, FDs...)
- A "last" chapter in the PhD of Laura Recalde, 1998



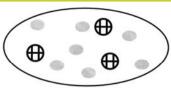
V. The Int. Conf. on Petri Nets and Related Models, Miami, 2005

- Continuisation of Timed Petri Nets: From Performance Evaluation to Observation and Control (inv. lecture)
- We (re)comment on "fluid views"
- Molecules vs mols:
 - Fractions of moles
 - He found "not surprising" my (first) degree on Chemicalengineering ©



Systems are NOT...:

- we always manage models: "views" of systems
- are we able to move form one to another level? (i.e., when are we able? / how? / ...)



Predator/prey: Volterra-Lotka equations...

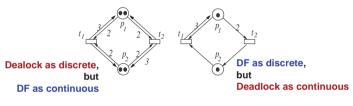
Manufacturing engineers, sometimes seems to be hydraulic engineers

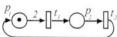


On Unfluidisable Models



Deadlock-freeness: discrete vs continuous (?)





7FNO

Liveness: yes or not?: lim-reachability (!)

On fluid / continuous PNs



if there is no dead transition (no siphon unmarked):

- all the potential repetitive sequences are realizable
- Behavioural and structural synchronic relations coincide (in particular bounded= structurally bounded)
- Therefore synchronic relations can be computed in polynomial time!!

For hybrid and discrete net systems, the above is not necessarily true (usually false)

timed models: infinite server semantics-minimum



Infinite Servers Semantics (variable speed)

$$\mathbf{f}(\tau)[t] = \lambda[t] \cdot \text{enab}(t, \mathbf{m}) = \lambda[t] \cdot \min_{p \in {}^{\bullet}t} \left\{ \frac{\mathbf{m}[p]}{\mathbf{Pre}[p, t]} \right\}$$

Firing rate: proportional to the input "level" Analogous to the discrete "markovian" case

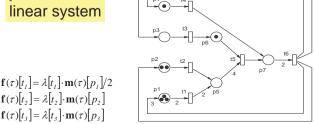
A set of *switching* differential linear systems
$$\begin{cases} \mathbf{m}(\tau) = \mathbf{C} \cdot \mathbf{f}(\tau) \\ \mathbf{f}(\tau)[t] = \lambda[t] \cdot \min_{p \in {}^{t}t} \left\{ \frac{\mathbf{m}[p](\tau)}{\mathbf{Pre}[p,t]} \right\} \end{cases}$$

example:

infinite server semantics-minimum



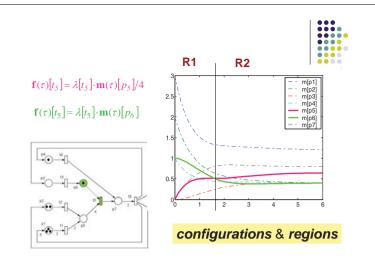
piecewise linear system



 $\mathbf{f}(\tau)[t_3] = \lambda[t_3] \cdot \mathbf{m}(\tau)[p_3]$ $\mathbf{f}(\tau)[t_A] = \lambda[t_A] \cdot \mathbf{m}(\tau)[p_A]$

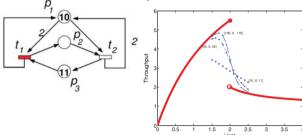
 $\mathbf{f}(\tau)[t_5] = \lambda[t_5] \cdot \min \{\mathbf{m}(\tau)[p_5]/4, \mathbf{m}(\tau)[p_6]\}$

 $\mathbf{f}(\tau)[t_6] = \lambda[t_6] \cdot \mathbf{m}(\tau)[p_7]/2$



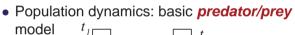
one of the counterintuitive properties in timed (infinite servers semantics) models

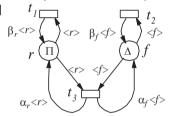
- No performance monotonicities with respect to firing speeds
- Discontinuities, bifurcations



infinite number of switchings Continuous PNs under infinite servers semantics simulate TURING Machines:

decolouring and net interpretation-product

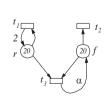


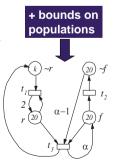


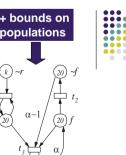
- rabbits
- foxes

Consequende of product-semantics: it is possible to simulate the Lorentz's equation.... chaos !!!

decolouring and net interpretation-product



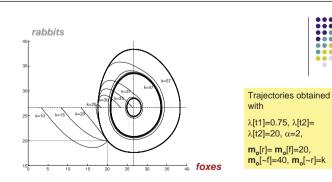




P/T models of the predator/prey system, with $\alpha_r=0$, $\alpha_f=\alpha$, $\beta_r=2$, $\beta_f=0$, $|\Pi|=|\Delta|=20$. They can be seen either as discrete or as continuous PN.

$$\begin{cases} \overset{\bullet}{r} = \lambda_1 \cdot r - \lambda_3 \cdot r \cdot f \\ \overset{\bullet}{f} = \lambda_3 \cdot (\alpha - 1) \cdot r \cdot f - \lambda_2 \cdot f \end{cases}$$

synchronisations leads to *products*



Discrete	Continuous	
(Stochastic)	(Deterministic)	
(Un)bounded	Bounded	
Non-live	Live	

Concluding the selected points ..

- As a difference with other experiences, usually my conversations with Carl Adam Petri were not really long, I try to keep on very concrete things.
- We should remember Carl Adam as:
 - the founder of a field, that influence others, that combine conceptual elegance & pragmatism;
 - a close & warm-hearted person.

...and to conclude with an image-souvenir...



